

**4.10 Predicting the Number of Roots of Quadratics**

Recall: For a quadratic relation  
roots = zeroes = x-intercept = solutions

Given vertex form, look at:

- the location of the vertex (above/below x-axis?)
- the direction of opening (up/down?)

Ex.1  $y = 3(x - 5)^2 - 1$

V( 5, -1 )

The vertex lies above/below? the x - axis.

The parabola opens up/down?

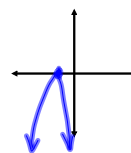
# zero(es): 2

Ex.2  $y = -2(x + 1)^2$

V( -1, 0 )

The vertex lies on the x-axis

# zero(es): 1



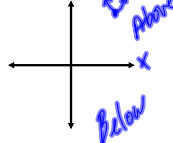
Ex.3  $y = 3(x - 5)^2 + 6$

V( 5, 6 )

above/below? the x-axis

opens up/down?

# zero(es): 0



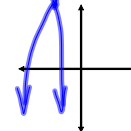
Ex.4  $y = -0.5(x + 3)^2 + 11$

V( -3, 11 )

above/below? the x-axis

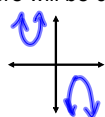
opens up/down?

# zero(es): 2



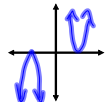
In general to identify the zeros from vertex form:

There will be 0 zeroes if the vertex is above the x-axis  
and the parabola opens up

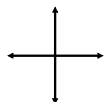


**OR** if the vertex is below the x-axis  
and the parabola opens down

There will be 1 zero if the vertex is on the x-axis



There will be 2 zeroes if the vertex is above the x-axis  
and the parabola opens down



**OR** if the vertex is below the x-axis  
and the parabola opens up

What if the quadratic is in standard form?

1. Factor and find the number of roots directly.
2. Complete the square (vertex form) and deduce the number of roots by visualizing the graph.
3. Use the quadratic formula if:
  - it cannot be factored
  - the numbers are too difficult to work with

Ex.1 Use the quadratic formula to determine the zeroes.

(a)  $-2x^2 - 4x - 2 = 0$   $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$X = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(-2)}}{2(-2)}$$

$$X = \frac{4 \pm \sqrt{0}}{-4}$$

# root(s): 1  
root(s): -1

(b)  $y = 3x^2 - 30x + 74$

$$X = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(74)}}{2(3)}$$

$$X = \frac{30 \pm \sqrt{12}}{6}$$

$$X_1 = 5.58 \quad X_2 = 4.42$$

# root(s): 2  
root(s): 5.58, 4.42

(c)  $y = 3x^2 - 30x + 81$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-(-30) \pm \sqrt{(-30)^2 - 4(3)(81)}}{2(3)}$$

$$X = \frac{30 \pm \sqrt{-72}}{6}$$

Can not square root a negative number

# root(s): 0  
root(s): none

$D = b^2 - 4ac$  is called the discriminant.

It tells you how many zeros the quadratic has.

D is positive  
 $b^2 - 4ac > 0$

two real roots

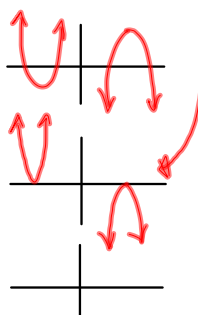
D is zero  
 $b^2 - 4ac = 0$

one real root  
(double root)

D is negative  
 $b^2 - 4ac < 0$

no real roots

$$y = a(x-h)^2$$



Ex.2 Find how many zeros each of the following quadratic relations has using the discriminant.  $D = b^2 - 4ac$

(a)  $y = x^2 - 6x + 7$   $D = (-6)^2 - 4(1)(7) = 36 - 28 = 8$   $\therefore$  2 distinct roots

(b)  $y = 2x^2 - 5x + 9$   $D = -47$   $\therefore$  no root

(c)  $y = x^2 + 3x - 11$   $D = 53$   $\therefore$  2 distinct roots

(d)  $y = 9x^2 - 24x + 16$   $D = 0$   $\therefore$  1 real root

Assigned Work: p.350 # 2, 3, 4, 5, 7, 9, 10, 12